MATH 54 – HINTS TO THE NOT-HOMEWORK

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Here are a couple of hints to the Not-Homework! (in case you're interested) Enjoy :)

Section 7.1: Diagonalization of Symmetric matrices

7.1.1. A is symmetric if $A^T = A$. In other words, the matrix is symmetric with respect to its main diagonal!

7.1.7, 7.1.13. Remember orthogonal matrices have **orthonormal** columns! Also, if A is orthogonal and square, then $A^{-1} = A^T$, so A^{-1} is easy to calculate!

7.1.13, 7.1.17. First diagonalize the matrix as usual, and then apply Gram-Schmidt **to each eigenspace**! Don't do it on all 3 eigenvectors, just do it on the eigenvectors on each eigenspace, so in practice you would have to do $2-3$ mini G-S processes instead of one huge one!

7.1.25.

(a) **T** (If $A = PDP^T$, then $A^T = PD^TP^T = PDP^T = A$)
(b) **T** (If A is symmetric, then eigenvectors corresponding to distinct eigenvalues are orthogonal)
(c) **F** (Consider $A = I$, it has only one eigenvalue $\lambda = 1$, but it is symmetric)
(d) **T**

7.1.26.

(a) **T** (This is theorem 2 on page 390)
(b) **T** (If $B = PDP^T$, then $B^T = PD^TP^T = PDP^T = B$)
(c) **F** (those are two separate concepts! Orthogonally diagonalizable means that there is an orthogonal matrix P such that $A = PDP^T$. For example, $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal, but not orthogonally diagonalizable because it is not symmetric! See theorem 2)
(d) **T** (that's another way of saying that symmetric matrices are diagonalizable)

7.1.29. If $A = PDP^T$, then

$$A^{-1} = \left(P^T\right)^{-1} D^{-1}P^{-1} = \left(P^{-1}\right)^T D^{-1}P^{-1} = \left(P^T\right)^T D^{-1}P^T = PD^{-1}P^T$$

Hence A^{-1} is orthogonally diagonalizable! Here we used the fact that $P^{-1} = P^T$ since P is orthogonal and square!

Or you can show that A^{-1} is symmetric using the fact that A is symmetric:

Date: Never.

$$\left(A^{-1}\right)^T = \left(A^T\right)^{-1} = A^{-1}$$

7.1.30. Here it is easier to use theorem 2. Just show BA is symmetric! But:

$$(BA)^T = A^T B^T = AB = BA$$

Hence BA is symmetric! Here we used the fact that $A^T = A$ and $B^T = B$ since A and B are symmetric and that $AB = BA$